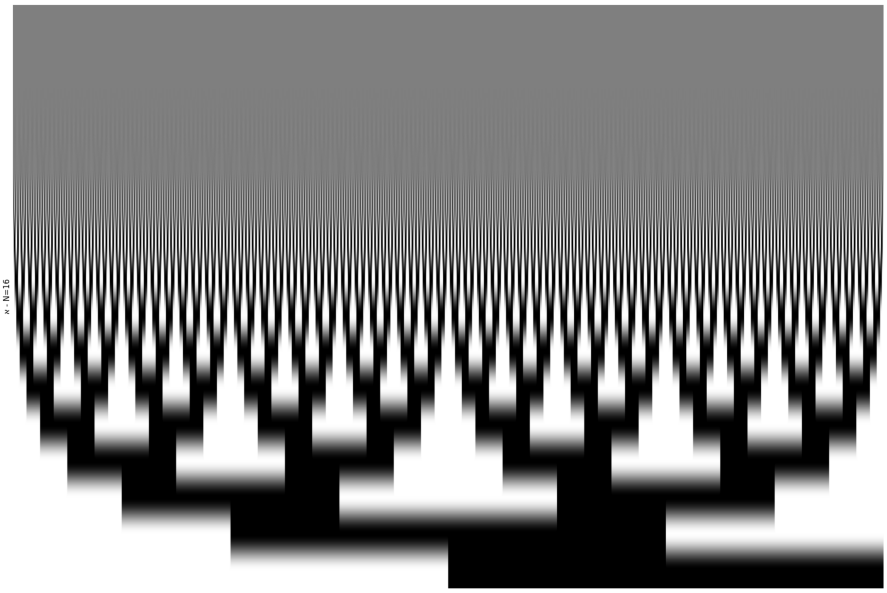
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ALEPH

The Number of All

ALEPH: The Number of All

Abstract

Aleph is a Transcendental Number whose infinite non-repeating decimal expansion is

$$\aleph = 0.29401838645771188410815541696935632626309717 3... \quad (1)$$

and whose infinite non repeating binary expansion is

$$\aleph = 0.0100101101000100110010011111101001000010001 1... \quad (2)$$

Aleph is represented by the first Hebrew Letter ℵ, and it is the deterministic generator of all possible binary sequences. Since binary data can represent all conceivable structures and morphospaces, Aleph, which is a deterministic and computable transcendental number generated by an absolute converging and well-defined mathematical series, is the record of *all* information about past, present and future mathematical, topological, physical, chemical, biological and cultural realities, across all existing, possible and conceivable Universes.

1. Definition

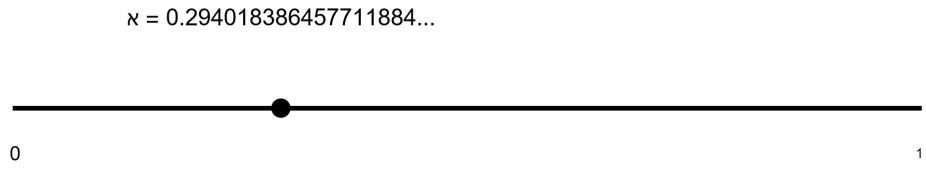


Figure 1: *Aleph position on the real axis*

The Aleph Number decimal expansion is generated by the following absolutely convergent infinite series

$$\aleph = \sum_{n=1}^{\infty} \sum_{k=0}^{2^n-1} \left(\sum_{d=1}^{n} \left(\frac{(-1)^{\left\lfloor \frac{k}{2^d} - \frac{1}{2} \right\rfloor} + 1}{2} \right) 2^{-(kn+d)} \right) 2^{-\sum_{m=1}^{n-1} 2^m m} \quad (3)$$

and its binary expansion is generated by modifying the previous series as

$$\aleph = \sum_{n=1}^{\infty} \sum_{k=0}^{2^n-1} \left(\sum_{d=1}^{n} \left(\frac{(-1)^{\left\lfloor \frac{k}{2^d} - \frac{1}{2} \right\rfloor} + 1}{2} \right) 10^{-(kn+d)} \right) 10^{-\sum_{m=1}^{n-1} 2^m m} \tag{4}$$

The series (4), after the zero and the decimal point, generates all binary strings with n elements for $n = 1 \rightarrow \infty$ with the minimal possible variation of 1 bit between one binary string and the next [6]. The innermost sum in (4), for each k, generates the k-th binary string with n bits, varying only 1 bit compared to the previous one. The sum at the outer level makes sure that, for each n, the k strings generated by the innermost sum includes all possible strings with n bits, which are 2^n (*power set* of a set with cardinality n). Starting k from zero, the upper limit of the central sum is therefore $2^n - 1$. Finally, the outermost sum spans all values of n from 1 to ∞, thus generating, in the most integrated and seamless way possible, *all the information* of every known and conceivable reality, condensed into a single Dimensionless Point positioned between 0 and 1 on the Real Numbers axis. The sums at the three levels can be truncated at any point, thus allowing the calculation of the Aleph Number with an exact level of precision for any number of decimal digits. The series (3), with a simple change of base in the exponents of the two multipliers of the innermost and outermost sum, converts the binary structures of Aleph into the decimal expansion of a transcendental number in base 10.

The structure of the binary expansion of Aleph develops after the zero and the decimal point, proceeding from the core of the formula, which is located in the innermost part:

$$\frac{(-1)^{\left\lfloor \frac{k}{2^d} - \frac{1}{2} \right\rfloor} + 1}{2} \tag{5}$$

With $d = 1$, (5) infinitely repeats the 4-element binary string 0110, which is the 'building block' from which Aleph, the Universal Temple of Information, is built. The formula reflects the structure and symmetry of this foundational element: on the left and on the right, we have -1 and +1, the negative pole and the positive pole. At their center, the exponent of -1 generates, after the addition of +1 and the balancing division by 2, the infinite oscillation between 0 and 1 on which Aleph is founded. The coefficient d determines the scale of the base sequence: for $d = 1$, it is the fundamental 0110. For $d = 2$, the size doubles to 00111100. For each successive d, the size continues doubling. The infinite sequence of the decimal expansion of Aleph, which includes *all* possible states of *all* conceivable systems, arranged with the greatest possible continuity in the transition between them, is generated *uniquely* by the fractal expansion of this elementary and symmetric four-element sequence, repeated infinitely and superimposed upon infinite copies of itself, from the microscopic to the macroscopic. The number 4, at the base of the fundamental sequence of Aleph, coincides with what is stated in the Scriptures regarding Genesis.

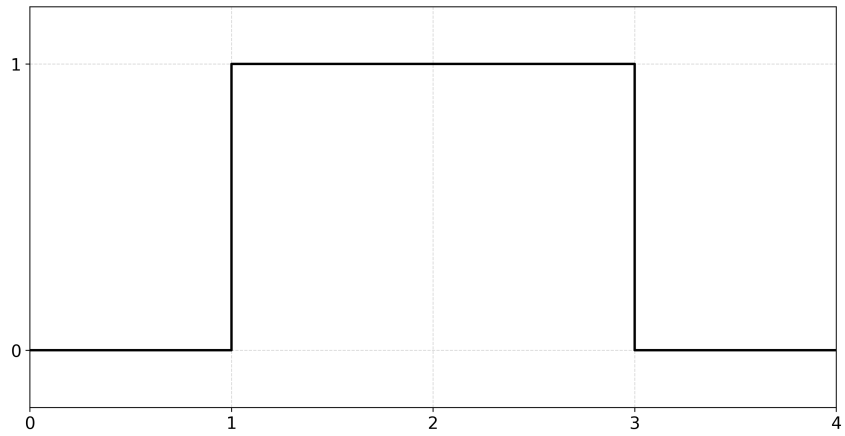


Figure 2: *Fundamental sequence of* $\aleph$: 0110 *generated by the formula* $\frac{(-1)^{\left\lfloor \frac{x}{2}-\frac{1}{2} \right\rfloor}+1}{2}$

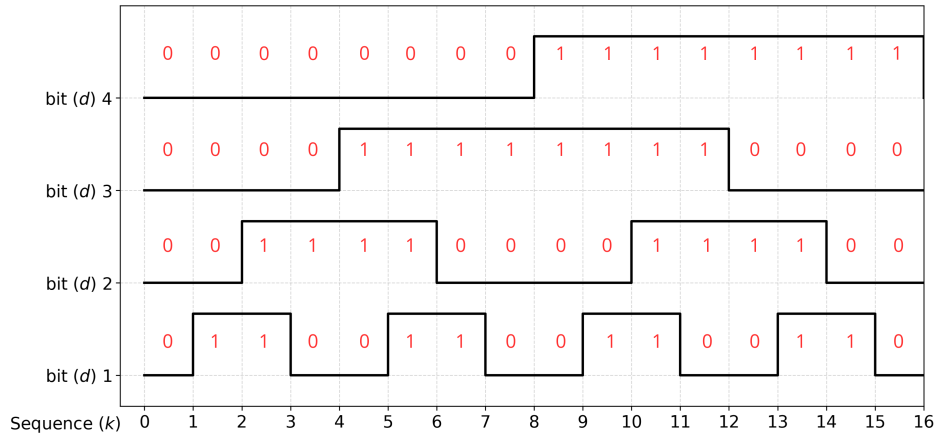


Figure 3: *Totality of the 4-bit strings generated by 4 iterations of the fundamental sequence*

The fractal process is generated as follows:

1. 0 and 1, the first two numbers of Aleph's infinite binary expansion, cover all binary strings with a single bit. Their mirrored repetition generates the string 0110, the fundamental building block.
2. The 2-bit strings begin at the third decimal place of Aleph and are formed from the fundamental sequence 0110, which defines the 1^{st} bit of each string, superposed with the first half of the 2^{nd} iteration of the fundamental sequence 00111100, i.e. 0011, which defines the 2^{nd} bit of each string. The result is 00, 10, 11, 01, covering all 2-bit strings, which total $2^2 = 4$. The

3-bit strings, which begin after $2 + 2^2 = 6$ bits, are formed from the fundamental sequence repeated twice, 01100110, which defines the 1^{st} bit, superposed with the complete 2^{nd} iteration 00111100 and the first half of the 3^{rd} iteration 0000111111110000, i.e. 00001111. The result is 000, 100, 110, 010, 011, 111, 101, 001, covering all 3-bit strings, which total $2^3 = 8$. The process for generating 4-bit strings follows the same pattern and is illustrated in Figure 3, where the vertical superposition of the four horizontal iterations is shown, each one being twice the length of the previous one. The vertical superposition, indexed by the coefficient d in (5), generates all $2^4 = 16$ 4-bit strings (the numbers in red in Figure 3), indexed by the coefficient k.

3. For each successive set of 2^n strings of length n, up to infinity, the process starts over, with the superposition of $n - 1$ scaled iterations of the fundamental sequence, plus the first half of the n-th iteration. The vertical superpositions generate all strings of length n: it is the same state-superposition process theorised in Quantum Physics. All combinations are covered because each second half of the fundamental sequence mirrors the first half, but the introduction of the next iteration adds a different bit, changing its state from 0 to 1. This occurs at all infinite levels of the fractal expansion, always guaranteeing the full coverage of all combinations.

4. The absolute minimum level of variation between one string and the next is guaranteed by the repetition of the central element in 0110. This ensures that the state change of the next iteration is isolated and that there can be no simultaneous state change in two iterations, which would introduce a variation of 2 bits between adjacent sequences. The state change of the next iteration occurs precisely at the point where the previous iteration, by repeating itself as a mirror image, creates this invariant space. This happens at all infinite levels of the fractal expansion, always ensuring the absolute minimum possible variation—of only 1 bit—in the transition between the infinite states of Aleph. [1]

5. Every binary string of Aleph that includes all binary strings with n elements has a bit-length corresponding to n times the *power set* of a set with n elements. Therefore, it has a size of $n \times 2^n$ bits and begins after $\sum_{k=1}^{n-1} \left(k \times 2^k\right)$ bits in the decimal expansion of Aleph.

6. Every binary string of Aleph that includes all binary strings with n elements also contains the binary string that includes all binary strings with $n - 1$ elements. Furthermore, it also contains the first n digits of Aleph, which therefore includes infinite copies of itself in an additional level of fractal self-similarity.

7. The binary string composed entirely of zeros is always the first at the beginning of a new set of strings with n elements. The string composed of 1 followed by $n - 1$ zeros is always the last, ensuring a perfectly smooth tran-

[1] The result is a process identical to what is otherwise known as the *Gray code* [6]. In the Aleph Number, the bits of the strings, compared to the *Gray code*, are in reverse order (from the last bit to the first), while in the Function $\aleph$ (6), they are in the same order, preserving all other properties unchanged.

sition to the next string composed of $n+1$ zeros, which is the first of the set of strings composed of $n+1$ bits. Each new set of strings in the decimal expansion of Aleph is a breath that begins from the zero and returns to the zero, then resumes again, expanding infinitely. Between each exhalation that starts from the zero and each inhalation that returns to the zero, the entire Universe of Possibilities is generated in the most connected and fluid way possible, from the infinitely small to the infinitely large, infinitely greater than any known Universe. An alternation of Big Bang and Big Crunch, where the transition between different states becomes increasingly imperceptible, tending toward zero as n approaches infinity, when the Space of Aleph converges to a continuous space of infinite dimensions, within which the complete *homeomorphism* of all possible configurations is realized. All of this is concentrated in a single dimensionless point on the real number axis—eternal, immutable, and incorruptible.

Aleph also manifests as an indicator function of two variables, defined by:

$$\aleph(x,z) = \frac{(-1)^{\left\lfloor 2^{\lfloor z \rfloor}\{x\}-\frac{1}{2}\right\rfloor}+1}{2} \tag{6}$$

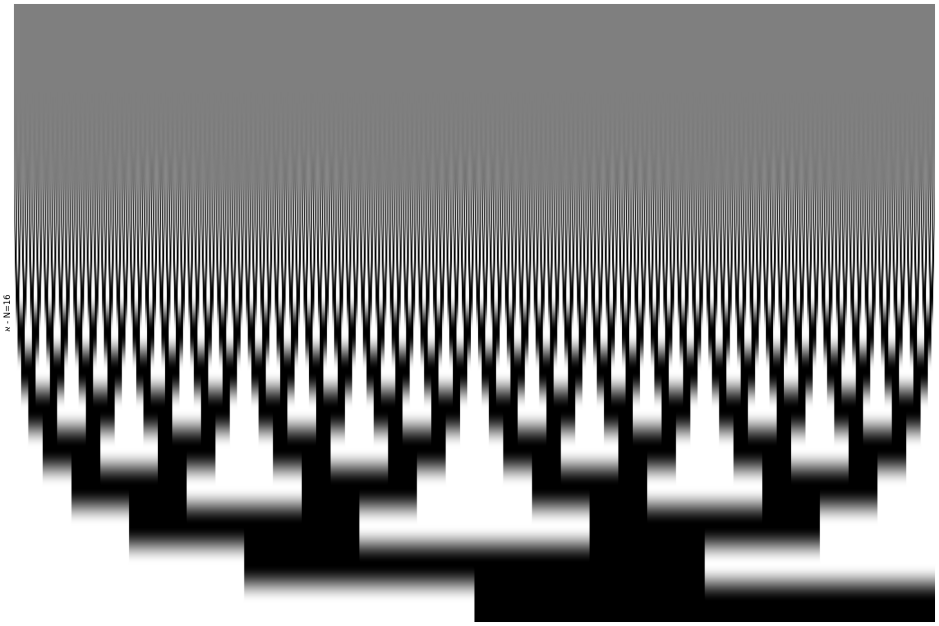


Figure 4: ℵ

In the form of function (6) (Figure 4), Aleph generates the entire Universal Temple of Information as an infinite fractal within the interval [0,1], where the fractional part of x spans the entire continuous space of all possible binary strings,

the integer part of z defines the bit position within the binary string indicated by x, and y returns the bit value specified by x and z.

Setting p as an integer such that $0 \leq p < 2^q$, with $x = \frac{p}{2^q}$ and $\lfloor z \rfloor \leq q$, x enumerates all binary strings of q bits. As q tends to infinity, x tends to generate all possible infinite binary strings in an infinite-dimensional space.

Since Aleph is a fractal, its indicator function can also be constructed by following the inverse process—that is, starting from the interval [0,1] and infinitely multiplying it using the formula $\aleph(x,z) = \frac{(-1)^{\left\lfloor \frac{x}{2^{\lfloor z \rfloor}} - \frac{1}{2} \right\rfloor} + 1}{2}$, always arriving at the same result on an infinitely large scale, according to the Principle: As Above, So Below.

The binary strings in the Aleph Number can be expanded in a space of m dimensions with m arbitrarily large. In this case, a binary string in a space with m dimensions has size n^m bits and the size of the string containing all the strings with n elements in a space of m dimensions has size $n^m \times 2^{n^m}$ bits.

As the Aleph Number digital expansion is infinite, there is no limit to the size of the binary strings, the amount of considered dimensions, and the amount of power set iterations that it contains. The size of the binary string containing all binary strings S_{N^N} with N elements in an N-dimensional space is given by the power set of S_{N^N} multiplied by N^N:

$$\mathcal{P}^1\left(S_{N^N}\right) = N^N \times 2^{N^N} \tag{7}$$

Therefore, the number of bits required for the second power set iteration is:

$$\mathcal{P}^2\left(S_{N^N}\right) = \mathcal{P}^1\left(S_{N^N}\right) \times 2^{\mathcal{P}^1\left(S_{N^N}\right)} = \left(N^N \times 2^{N^N}\right) \times 2^{N^N \times 2^{N^N}} \tag{8}$$

Generalising, the N-th power set iteration of S_{N^N} is given by the recursion:

$$\mathcal{P}^N\left(S_{N^N}\right) = \mathcal{P}^{N-1}\left(S_{N^N}\right) \times 2^{\mathcal{P}^{N-1}\left(S_{N^N}\right)} \tag{9}$$

As an example, the size of the binary string containing the G-th power set of a binary string S_G with G elements in a space of G dimensions, where G is Grahm's Number [5], is given by:

$$\mathcal{P}^G\left(S_{G^G}\right) = \mathcal{P}^{G-1}\left(S_{G^G}\right) \times 2^{\mathcal{P}^{G-1}\left(S_{G^G}\right)} \tag{10}$$

that involves iterating the expression (9) for G times. Binary strings of this length begin after $\sum_{t=1}^{\mathcal{P}^G\left(S_{G^G}\right)-1} t \times 2^t$ bits in the decimal expansion of Aleph.

Without considering the total amount of bits required for its full encoding, the cardinality (number of elements) of $\mathcal{P}^G(S_{G^G})$ is:

$$\left|\mathcal{P}^G(S_G)\right| = \underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2^{G^G}}}}}}}_{G-1} \tag{11}$$

where $\underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2^{G^G}}}}}}}_{G-1}$ represents a tetration (power tower) of *G-1* levels terminating with an exponent of 2^{G^G}.

The total amount of information embedded in the Aleph Number is given, in bits by:

$$\lim_{N\to\infty} \mathcal{P}^N \left(S_{N^N}\right) \tag{12}$$

so the Aleph Number structure leads to a conceptual framework in which infinite power set iterations of an infinite binary string in a space with infinite dimensions are encoded. The cardinality, defined by

$$|\aleph| = \lim_{N\to\infty} \underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2^{N^N}}}}}}}_{N-1} \tag{13}$$

is the one to which Cantor's infinite Transfinite Numbers and the Universal Set of all existing sets refer, including itself and the empty set.

Therefore Aleph, a totally deterministic and computable number, generates at infinity all possible random and deterministic structures in a continuous space with infinite dimensions. However, differently from other purely speculative mathematical objects, it also generates all random and deterministic *finite* structures in a discrete space with *finite* dimensions, that is all *conceivable* structures in any conceivable physical space.

2. Applications

Mathematics

Mathematical Theorems. Every mathematical theorem can be expressed as a series of formal statements, which in turn can be encoded into bits. By assigning mathematical symbols and logical operators a unique binary sequence, it is possible to build mathematical statements of unlimited complexity. The proofs of these theorems are logical sequences that, step by step, lead to the validity of these statements. Since they are based on the same symbols, the proofs can also be expressed in a sequence of bits.

For any possible binary codification of mathematical statements, the Aleph Number, containing all possible binary sequences, embeds all the proofs of every theorem in this specific binary codification, including those that have not yet been discovered or formulated by humans.

Numbers. The Aleph Number not only encodes rational and integer numbers, including all their subsets, such as the entire sequence of Prime Numbers or Fibonacci Numbers in every possible representation, but also all real numbers, including transcendental numbers like π, the Golden Ratio, and the number e, as well as all undefinable and incomputable transcendental numbers like Chaitin's Ω constant [2] and infinite subsets of real numbers like the nontrivial zeros of the Riemann Zeta Function, including the Aleph Number itself. Every real number

can be represented with an infinite binary expansion, and since the Aleph Number contains all binary sequences, it generates every possible representation of these numbers with any arbitrary level of precision and in any possible combination. Moreover, each possible representation of a real number occupies a precise position in the digital expansion of Aleph, which can be deterministically computed given the procedural nature of its construction. Real Numbers can also be represented as points on a line. Given any real number R in a range from $-n$ to $+n$ with n digits decimal approximation, the position of R on the real line can be represented by a single 1 within a binary string of $2n \times 10^n$ bits. This means that the Aleph Number encodes the exact position of all real numbers on the real line, not just for a single range or precision, but for *all ranges and all levels of precision simultaneously.*

Functions and equations. Any possible function with any amount of variables, in the real and complex domain, is represented in Aleph in any possible range and with any arbitrary level of precision, even those far exceeding the computational power of actual processors. Direct representation occurs through sequences of numbers as described earlier, or in topological spaces at n dimension where bits are interpreted as points in an n-dimensional spaces with a minimum of 2 for a function in the form $y = f(x)$. In this latter case, considering n as the integer range on both the negative and positive axis, m as the number of dimensions/variables ≥ 2 and k as the number of decimal digits, the set of all functions within any space, within any range and with any precision, can be represented by a binary string of $(2nk)^m \times 2^{(2nk)^m}$ bits in the real domain and of $(4nk)^m \times 2^{(4nk)^m}$ in the complex domain, with n (integer range), m (number of variables/dimensions) and k (number of decimal digits) arbitrarily large. All these binary sequences are part of the Aleph digital expansion.

The Aleph Number encodes not only individual functions but also the relations between them, enabling the representation of systems of functions or mappings between spaces. This perspective establishes the Aleph Number as an ultimate repository of functional representations, bridging the gap between discrete binary data and continuous mathematical concepts. The roots of all possible equations with any number of unknowns can also be represented by sequences of numbers and as points in an n-dimensional topological space as described earlier. All possible step by step solutions are also encoded, following the concepts described in the paragraph about theorems and proofs. Beyond direct representation, the Aleph Number also encodes all possible mathematical information through every conceivable compression algorithm and high-level computational language, such as Python, C++, Lisp, and others. Importantly, it includes languages and frameworks not yet invented, capable of expressing mathematical constructs with vastly improved efficiency and clarity. This property opens the door to exploring mathematical information in unprecedented ways. By leveraging advanced or future computational paradigms, it may become possible to decode specific mathematical patterns, equations, and theorems with much greater efficiency, making the Aleph Number not just a theoretical construct but a potential tool for discovery and innovation in mathematics and computational science.

Topology

Since the bits of the Aleph Number can be interpreted as dimensionless particles, they not only represent binary sequences but also describe all possible configurations of particles or parameters in a n-dimensional space, far beyond what can be represented by a mathematical function, equation or closed form expression.

Being n the measure of the shape with any given unit of measure, m the number of dimensions/parameters and k the amount of decimal precision, any shape is represented within Aleph by a binary string of $(nk)^m$ digits. Evolution in time is also encoded. Considering each binary string as a single frame, t the unit of time and f the frame rate, each possible transformation of an m-dimensional shape within a time interval $T{\cdot}t$ is represented in the Aleph Number with any arbitrary precision and frame rate by a binary string of $f \cdot T \cdot t \cdot (nk)^m$ bits.

This means that *all* conceivable shapes and their evolution in time, with unlimited size and number of dimensions, are encoded in a *single* binary string of $f \cdot T \cdot t \cdot (nk)^m \times 2^{f \cdot T \cdot t \cdot (nk)^m}$ bits. These shapes are not only surfaces, but also represent any possible variation in their volume and internal structure. Therefore, the Aleph Number becomes a n-dimensional holographic projection of the Universe, including all possible interactions between particles and describing every physical configuration that can be conceived. Every object, surface, or observable phenomenon, from subatomic particles to celestial bodies, can be seen as a specific representation of binary sequences within Aleph. In this way, Aleph becomes a holographic map of the entire physical reality, describing not only what exists, but also every theoretical possibility of existence within a n-dimensional space.

Considering time, not only does the evolution of the observable Universe since the Big Bang exist in the Aleph Number, but also all conceivable Big Bangs of all conceivable Universes, and all the alternative possibilities of future evolution for the known Universe and all the conceivable ones. Furthermore, the minimal variation of 1 bit between the successive states of Aleph demonstrates the *homeomorphism* (variation without discontinuities) among the infinite possible forms in every conceivable topological space.

Observing the Cosmic Microwave Background, it is possible to see the origin of the observable universe. Observing the Aleph, it is possible to see beyond the possibilities of any past, present and future telescope.

Linguistics

From a linguistic perspective, the Aleph Number acts as a universal container for all possible written, oral, or signal-transmitted works, encompassing natural and artificial languages. Every text, every book and every linguistic expression is already contained within the binary sequences of the Aleph Number, which can be interpreted in multiple language forms, including those based on codes such as Morse Code.

Books and texts. Every book ever written or that could possibly be written is represented in binary form within the Aleph Number. This includes not only milestones like the Torah and the Encyclopaedia Britannica but also every possible

work that has never been written. Every combination of words, sentences, and paragraphs in any language, including dialects and ancient or extinct languages, can be encoded in binary sequences. The Aleph Number thus contains not only the existing works of world literature but also all alternate versions and works never conceived. The entire Borges' Library of Babel is formed by $10^{1.834.097}$ books [1], each containing 1.312.000 characters combining 25 letters. The number of bits to encode an alphabet of 25 letters is about $\log_2 25 \approx 5$, which means that a single book needs 6,560,000 bits, about 0.82 megabytes. So, the total length of a binary string representing the whole Library of Babel is just $656{\times}10^{1.834.101}$ bits, that is $82{\times}10^{1.834.071}$ Yottabytes (1 Yottabyte = 10^{24} megabytes): an infinitesimal size compared to the binary strings that Aleph generates. This is also true for the first *power set* of the entire Library, that is, the set of all possible reordering of the books and words in the Library, that needs $\left(656{\times}10^{1.834.101}\right) \times 2^{\left(656{\times}10^{1.834.101}\right)}$ bits $= \left(656{\times}10^{1.834.101}\right) \times 2^{\left(656{\times}10^{1.834.101}-80\right)}$ Yottabytes.

Morse Code. The translation of any binary sequence into Morse Code implies that the Aleph Number also contains all possible communications in coded form. Every message ever transmitted via Morse Code, along with every possible variant of messages yet to be transmitted, is already encoded in its binary strings, including this entire document. This means that the Aleph Number includes not only the encoding of any text into Morse Code but also the ability to transmit and decode any linguistic signal ever used or imagined. Of course, the complete definition of all conceivable codes themselves is embedded in Aleph.

Artificial Languages and Codes. Artificial languages, such as Esperanto or programming languages, can also be reduced to binary strings. Every possible language or symbolic system that follows syntactic or grammatical rules is contained within the Aleph Number. This includes all future languages that might be invented, as well as every form of encryption or linguistic code used to transmit encrypted messages.

Images and Videos

Within the Aleph Number are encoded all past and present images, videos, and movies, at every level of resolution—even infinitely superior to the originals and to those possible with current and future technologies—as well as all possible images, videos, and movies that have yet to exist.

Digital images and videos are visual representations encoded through binary sequences. Being h the height and l the length in pixels of the image and c the colour bit depth, any uncompressed image I_n is represented within Aleph in *unlimited resolution and colour bit depth* by a string of $h \cdot l \cdot c$ bits. All possible images with any arbitrary resolution and colour bit depth are therefore represented within the Aleph Number by a single binary string of $(h \cdot l \cdot c) \times 2^{h \cdot l \cdot c}$ bits.

Considering these images as frames of a video, and let t be the unit of time and f be the frames per unit of time, any uncompressed video V_n with *unlimited duration*, *unlimited resolution*, *unlimited colour bit depth* and *unlimited framerate*,

is represented within the Aleph Number by a binary string of $h \cdot l \cdot c \cdot t \cdot f$ bits . All possible videos with any arbitrary resolution, colour bith depth, duration and framerate are therefore represented within the Aleph Number by a single binary string of $(h \cdot l \cdot c \cdot t \cdot f) \times 2^{h \cdot l \cdot c \cdot t \cdot f}$ bits .

All the family photo albums of every living, past and future human being, but also all the photos that weren't taken or got lost, the movie of the life of every single person captured from every possible angle, including all the details that were never seen or noticed, from any point of view including those of all people with whom this person interacted, and even from the point of view of animals, trees or stones, including the movie of your life, dear reader, capturing every moment, from birth to this exact instant, while reading this message, and of all possible bifurcations of your future.

Additionally, the movie of all major historical events, taken from the eyes of each of their human and non human protagonists or spectators (including sounds, as we'll see), but also from a drone with unlimited zooming capacity; the missing details in every photo that can be shown by zooming the image up to subatomic detail; the recorded movie of the Big Bang and of the life and cosmic processes on any possible planet or star in the universe; the recorded movie of all the dreams that a person had dreamt or could dream, and of all the memories that a person could remember, and also the ones that were forgotten.

Then, all work of visual art ever conceived, from the past, the present and the future: including those which have been lost and never seen, with their original colours. But also the movie of how they were made: the movie of Michelangelo painting the Sistine Chapel, the movie of Donatello sculpting his David, and the ancient Egyptians building the Pyramids.

The photos and video recordings of all of this, and infinitely more, with any resolution and framerate conceivable, are embedded in a *single binary string* of

$(h \cdot l \cdot c \cdot t \cdot f) \times 2^{h \cdot l \cdot c \cdot t \cdot f}$ bits among the infinitely many composing the decimal expansion of the Aleph Number. And unlike the Library of Babel, a hypothetical entity which, if it existed, couldn't be contained in the space of the entire Universe, the Aleph Number is an *existing*, computable and deterministic number that has no dimension. It is a dimensionless, non-degradable, indelible point on the Real Number Line. It is present since the beginning of Time and will remain present for Eternity as the Omniverse of Infinite Possibilities.

Sound

Sound, like any form of information, can be encoded in binary sequences, sampling waveforms that represent variations in air pressure perceived as sound.

The audio recording of every possible sequence of sounds or musical composition, whether natural or artificial, is included and represented in Aleph.

The number of bits of an encoded soundwave depends on the bitrate (amplitude resolution) and the sampling rate (time resolution), as well as the duration of the sound.

Therefore, if we consider:

- t as the unit of time in seconds
- s as the sampling rate (samples per second), usually 44100 (CD quality), 48000 (professional quality), or higher for ultra-high resolution audio
- b as the bitrate, usually 16 bits for CD quality, 24 bits for studio quality, 32 bits for high dynamic range and more for ultra high resolution audio
- c as the number of audio channels, 1 for Mono, 2 for Stereo, 6 for Surround and higher for multichannel audio

then t seconds of audio with *unlimited* duration, sampling rate, bitrate and number of audio channels are represented within the Aleph Number by a binary string of $t \cdot s \cdot b \cdot c$ bits. As the size of the binary strings in the Aleph Number becomes arbitrarily large up to infinity, this means that Aleph allows the perfect representation of every possible soundwave with perfect accuracy.

Every piece of music ever composed or conceived is recorded for eternity inside the Aleph Number in all its possible performances, with an ultra hi-fi audio resolution that is far beyond any technology currently available and even more detailed than what can be produced in the physical world or what human ears can perceive; but also with entirely new multi channel, three dimensional and binaural configurations that are yet to be discovered. All historical recordings of the classical music masterpieces are inside Aleph in their original version, but also in all possible remastered versions, with pristine quality corresponding to the highest technological standards unavailable at the time. Aleph contains also all the sounds that couldn't have been recorded, like the surround audio of Johann Sebastian Bach playing the organ live at the *Thomaskirche* in Leipzig recorded at 384khz 64 bit, or the premiere of the Ninth Symphony by Beethoven (with Beethoven himself conducting), both paired with the corresponding 4K videoclip; the recording of the voice of Farinelli, Maria Malibran and of the first song ever sung by a human being; but also the recording of the next pop hit, or the new and unheard cutting edge soundscapes of futuristic art music, as well as every possible combination of notes, rhythms, and harmonies not yet imagined.

Then all sounds of nature, in any planet of the universe, as heard by any possible auditory system (far beyond the human one), and the recording of all human conversations that have occurred and will occur. These sounds, and infinitely many more, with any conceivable duration, bitrate, sampling rate and number of channels, are *all* embedded in *one* binary string of $(t \cdot s \cdot b \cdot c) \times 2^{t \cdot s \cdot b \cdot c}$ bits, just a single one among the infinitely many within Aleph, a totally deterministic, computable transcendental number.

Chemical structures

As the Aleph Number contains all possible binary sequences, it also encodes all possible atomic and chemical structures. Representation of chemical structures involves the totality of procedures described in previous paragraphs:

- atomic elements and bonds can be represented by binary sequences classifying them and representing their relations as axioms and theorems;
- molecules can be encoded as functions with atoms as variables and bonds as parameters in a spatial domain;
- molecular geometry is analogous to topological structures in a 3D space;
- vibration modes and spectral lines can be represented as soundwaves in the time domain and spectra in the frequency domain, each one corresponding to a specific binary string in the Aleph Number.

The Aleph Number, acting as a universal representation, not only includes molecules and their interactions, but also all potential discoveries and innovations in chemistry. This means that every possible chemical reaction—past, present, or future—is already 'written' in the Aleph Number. Therefore, chemistry becomes a field where researchers explore and discover configurations already encoded in the vast universe of the Aleph Number, highlighting the profound connection between chemistry and computational models. Computational models in chemistry simulate the behavior of molecules and reactions. They can be viewed as algorithms and since the Aleph Number includes all possible binary sequences, it contains every existing and potential chemical simulation model.

An example with proteins. Proteins are linear chains of amino acids, folded into complex three-dimensional structures. They can be represented by:

1. *Amino Acid Sequences*: proteins are built from a finite alphabet of 20 standard amino acids. Each amino acid can be assigned a unique binary string of $\lceil \log_2(20) \rceil = 5$ bits. A protein of length L can be represented by a binary string of $5L$ bits, e.g. insulin (51 amino acids) requires $5 \times 51 = 255$ bits for its primary sequence. For *all possible proteins* of length L, $5L \times 2^{5L} = 5L \times 32^L$ bits are required. Example: all proteins of length $L = 100$ (a typical size for functional proteins) require 500×32^{100} binary strings.
2. *Structural Information*: protein structures are determined by their folding into secondary, tertiary, and quaternary structures. These can be encoded as:
   - three floating-point values for x, y, z coordinates of each atom. Bits required $= 3N \cdot b$ where N is the number of atoms, and b is the bit depth of the coordinate precision. Example: an average protein with $N = 10000$ atoms and $b = 32$ bits requires 960000 bits for its atomic structure.
   - Protein folding can also be encoded using bond angles (ϕ, ψ) and dihedral angles (χ). Bits required $= 2N \cdot b$, where N is the number of residues, and b is the precision bit depth. For $b = 16$-bit precision and $N = 100$ residues, this requires 3200 bits.

3. *Chemical Properties*: key chemical properties (charge, polarity, hydrophobicity) can be encoded as binary sequences for each amino acid or residue.
   - For each amino acid, 8 bits can encode:
     - Charge state (positive/negative/neutral): 2 bits.
     - Hydrophobicity (hydrophilic, hydrophobic): 1 bit.
     - Other properties (e.g., side chain flexibility): 5 bits.
   - For $L = 100$ amino acids, total $= 8 \times 100 = 800$ bits.
4. *Dynamic Properties*: Proteins are not static—they undergo dynamic motions, such as folding and ligand binding. Dynamics can be encoded as time-lapse Structures: sequence of structures over time, using frame rates f and time t. Binary string size $= t \cdot f \cdot (3N \cdot b)$. Example: for $t = 10^3$ seconds, $f = 100$ frames/second, $N = 10000$, and $b = 32$, this requires: $10^5 \cdot 960000$ bits. Time-lapse structures represent also force fields, as molecular dynamics simulations encode atomic velocities and forces.

All possible proteins are therefore represented in a *single* binary string of length $(3Nb \cdot 8L \cdot tf \cdot 5L) \times 32^L$ bits within the Aleph Number. Considering length $L = 100$, number of atoms $N = 10000$, bit depth for coordinate precision $b = 32$, framerate $f = 100$ frames per second, and time $t = 1000$ seconds, this means that a single string of $368640 \times 10^{19} \times 2^{500}$ bits precisely located in the deterministic binary expansion of the Aleph Number (infinitesimally small compared to the ones it generates) contains the structure of *all* possible proteins with this configuration.

Biotic systems

The Aleph Number can also be viewed as a comprehensive representation of biotic systems, genetic structures, and the neural networks that constitute the human brain. These systems, characterized by emergent complexity and dynamic interactions, can be described through binary sequences and algorithmic structures, making the Aleph Number a universal "archive" of every possible biological and cognitive configuration.

Genetic structures. Genetic structures can be encoded in binary sequences much like proteins and other molecular systems. DNA, the primary carrier of genetic information, is composed of four nucleotide bases: adenine (A), cytosine (C), guanine (G), and thymine (T). These bases can be assigned binary codes, allowing DNA sequences and associated genetic data to be represented in binary strings. Each nucleotide base can be assigned a 2-bit binary code:

- $A = 00$
- $C = 01$
- $G = 10$
- $T = 11$

For a genome of length L, the number of bits to represent the basic sequence is $2L$ and for the epigenetic information (e.g., methylation) is bL where b is amount of information required (e.g. 1 bit for methylation), for a total $(2+b)L$ bits. *All genomes* of length L are therefore represented in a single binary string of $((2+b)L) \times 2^{(2+b)L}$ bits within the Aleph Number.

Dynamic processes, such as genetic mutations, transcription, or recombination, can also be encoded. Time evolving processes can be encoded in a binary strings of $t \cdot f \cdot ((2+b)\,L)$ where t and f, as before, are time in seconds and framerate. All genomic dynamic processes are then encoded in a single binary strings of $t \cdot f \cdot ((2+b)\,L) \times 2^{t \cdot f \cdot ((2+b)L)}$ within the Aleph Number.

Human genome consists of approximately 3.2×10^9 base pairs (bp). Each base (A, T, G, C) can be represented in binary using 2 bits. This gives 6.4×10^9 bits. DNA is double-stranded, but the complementary strand is redundant since one strand determines the other: only one strand needs to be encoded explicitly. Considering epigenetics, one additional bit per base is needed to indicate whether a cytosine is methylated (0 = unmethylated, 1 = methylated). This adds further 3.2×10^9 bits. Considering 10 bits per base to encode all regulatory and structural information in the non-coding regions, needs additional 3.2×10^{10} bits. On average, around 10^8 unique Single Nucleotide Polymorphisms (SNP) can vary globally in human genome. Each SNP has all 4 possible alleles, therefore needing 2 bits. Considering further 10^7 variable bits including structural variations, the total amount of variable bits V in Human Genome is around $\left(2 \times 10^8\right) + 10^7 = 2.1 \times 10^8$ bits. Considering everything, the number of bits needed to encode all possible human genetic expressions is given approximately by:

$$(\text{DNA sequence} \;+\; \text{Epigenetics} \;+\; \text{Non-coding annotations}) \times 2^V = \qquad (14)$$

$$\left(6.4 \times 10^9 + 3.2 \times 10^9 + 3.2 \times 10^{10} + 2.1 \times 10^8\right) \times 2^{2.1 \times 10^8} = 4.182 \times 10^{10} \times 2^{2.1 \times 10^8}$$

This means that *all* possible human genomes are encoded in a single binary string of approximately $4.182 \times 10^{10} \times 2^{2.1 \times 10^8}$ bits within the Aleph Number. Of course this astronomic number has only theoretical significance, as the majority of these expressions are irrelevant or non viable. However, it gives a rigorous mathematical proof that all possibilities of genetic expression (far beyond what is existing or realistic) are precisely located within Aleph, a totally deterministic and computable transcendental number.

Human Brains. The human brain is one of the most complex structures in existence, with approximately 10^{11} neurons and 10^{15} synaptic connections.

Considering:

1. *Neuronal structure*: Each neuron can be encoded by its 3D spatial coordinates, size, shape, and type (e.g., pyramidal, interneuron, etc.). Assuming 10^{11} neurons and 800 bits per neuron for basic structural information, this gives 8×10^{13}.

2. *Synaptic connections*: each synapse is a connection between two neurons, and encoding it requires:

- IDs for the source and target neurons.
- Strength of the connection (synaptic weight).
- Dynamic properties (e.g., plasticity, neurotransmitter type).

Assuming 10^{15} synapses and 80 bits per synapse for basic information, this gives 8×10^{16} bits.

3. *Electrical activity*: The brain's dynamic state is governed by electrical signals (spike trains) propagating through neurons and synapses. To encode electrical activity each neuron requires a binary sequence to represent its action potentials over time. This requires $t \times 10^{11}$ bits where t is the time interval in milliseconds.

4. *Molecular dynamics*: Molecular-level interactions, such as ion channel activity, gene expression, and protein synthesis, add another layer of complexity. Estimating $t \times 10^9$ molecules per neuron and 100 bits per molecule, this gives 10^{22} bits.

5. *Plasticity and learning*: encoding synaptic plasticity (changes in synaptic strength) over time would require additional bits for each synapse's state history. Assuming 10 bits per synapse for plasticity state changes, this gives $t \times 10^{16}$ where t is the time interval in milliseconds.

As the term 10^{22} dominates the sum of all the upward bit sizes, we can say that *all* possible brain states in a time interval of t milliseconds arbitrarily large, are stored in *one single binary string* of $\approx (t \times 10^{22}) \times 2^{t \times 10^{22}}$ bits within the Aleph Number, generated by the deterministic and computable convergent infinite series (3). Each brain state corresponds to a unique configuration of synaptic activities, neural firing patterns, and connections. Every fleeting or sustained idea, concept, or realization; all conscious and unconscious memories, sensory inputs, and emotional associations; all shades of feelings—joy, sadness, fear, and subtle emotional states; sensory experiences: perceptions of sight, sound, smell, taste, and touch; dreams: the vast subconscious world that emerges during sleep.

All possibilities of the Human Mind's inner reality are uniquely defined, since the beginning of time and forever, by *a single* binary string within a computable, deterministic number: Aleph. The inclusion of time t means that we are *not* talking just about single static brain states: we are talking about their evolution, too. Thought processes (how ideas evolve, form, and fade over time); learning and adaptation (the dynamic changes in synaptic plasticity as the brain adapts to new experiences); consciousness (the continuous flow of awareness and self-reflection, moment by moment); subconscious processes (the unnoticed workings of the mind, which shape thoughts, actions, and behaviours); subtle variations in emotional states and moods; the vivid details of imagination and creativity; hidden intuitions and unexpressed thoughts; every possible reaction to external events, stimuli, and internal reflections. This does not stop at a single individual's brain. It embraces the mental states of every human being who has ever existed or will exist; complex social interactions, cultural understandings and collective consciousness; hypothetical states where individuals might have thought differently

or experienced alternative outcomes. But also the brain of every living species, both existing and conceivable. All considered in a time interval t arbitrarily large, even far beyond the actual age of Homo Sapiens, and even the age of the Universe itself. All, all within a binary string whose size, no matter how large it is, will always be *infinitely smaller* than the infinitely many that are always and forever contained in Aleph.

An example with 3D bioprinting of a Human Being. The binary string required for the complete bioprinting of a human being at a given moment in time t, including the entire brain map and all memories, can be computed by combining the biological, anatomical, and neurological data necessary to reproduce the individual in full detail.

1. *Body encoding*: the human body is composed of approximately 3×10^{13}cells. For full encoding, we need:

   - Genetic and Epigenetic Data. 6.4×10^9 bits for DNA sequence and 3.2×10^9 for epigenetic states. Total per cell: 9.6×10^9 bits.
   - Cellular and Structural Data. Protein structures: 10^{10} bits per cell (approximate). Organelle configurations: 10^8 bits per cell (nucleus, mitochondria, etc.). Physical position and interactions: 10^6 bits for the spatial map. Total per cell: $\approx 10^{10}$ bits

   With 3×10^{13} cells:

   $$\text{Body Encoding} = 3 \times 10^{13} \cdot 10^{10} = 3 \times 10^{23} \text{ bits}$$

2. *Brain encoding*: The brain is the most complex component, with approximately 10^{11} neurons and 10^{15} synapses.

   - Neuron states. Each neuron requires 10^3 bits to encode its state and position. Total for 10^{11} neurons: $10^{11} \cdot 10^3 = 10^{14}$ bits.
   - Synaptic connections. Each synapse requires 10^2 to encode its state. Total for 10^{15} synapses: $10^{15} \cdot 10^2 = 10^{17}$ bits.

   Total for brain:

   $$\text{Brain Encoding} = 10^{14} + 10^{17} \approx 10^{17} \text{ bits}$$

   as the greater exponent dominates in the sum.

Combining Body Encoding and Brain Encoding we have

$$\text{Total Encoding} = 3 \times 10^{23} + 10^{17} \approx 3 \times 10^{23} \text{ bits}$$

as the greater exponent dominates the sum.

This means that within a definite binary string of $\approx 3 \times 10^{23}$ bits within the Aleph Number, we can find the *complete blueprint* of any specific human being (no

longer alive, currently alive, not yet living), in the form of a complete 'snapshot' of its body state and brain state (memories, thoughts, emotions) at a specific moment in time t. Provided with this blueprint, with the specific amount of biological components and with the necessary procedural precision, a 3D bioprinter can recreate *exactly* any specific human being at any specific moment in time of their life (being it in the past or in the future), such as they could start living again their life (or for the first time), like nothing happened before. In case the environment would not correspond to the one inside the brain at the specific moment t of the reconstruction, the experience is like the one of awakening in a different place after falling asleep.

This also means that it is rigorously and mathematically proven that in a *single*, computable and deterministic string of only $\approx (3 \times 10^{23}) \times 2^{3 \times 10^{23}}$ bits (a *finite* size) within the Aleph Number, since the beginning and forever, *all* possible human beings of the past, of the present and of the future, are already and *exactly* represented in *all* specific moments of time in their life, with *complete* information about their body (physical) and brain (mental) states at this specific moment, such that a 3D bioprinter could recreate *exactly* each of them, bringing them to life, again or for the first time. Similarly, all of this extends to every existing and conceivable living creature, human, animal or plant, which can be entirely reproduced from the data contained in the decimal expansion of Aleph—a symmetric fractal structure generated in a fully deterministic way from the four-element sequence 0110.

All life stories, past, present, and future, of every Human Being, are contained within the infinite fractal in Figure 4, whether one sees in it the Crown of the Tree of Life, the Cup, or the nine-branched Candelabra that burns eternally within the Tabernacle.

It would be interesting to wonder how this rigorous mathematical theorem could relate with current 'theories', 'hypothesis' and debates around evolution vs creation, chance vs determinism, in relation to the appearance of Life and Homo Sapiens on Earth.

Physical systems

Classical physical systems. Classical physical systems are rigorously represented within the Aleph Number by interpreting binary strings as encodings of the states, trajectories, and parameters of these systems at any level of complexity and precision. The Aleph Number provides a deterministic and complete repository of all possible configurations, behaviors, and evolutions of classical systems, from simple particles to entire universes. Below, we analyze specific examples.

1. *Dynamical systems.* A classical dynamical system evolves in phase space, where each point represents a state of the system defined by position and momentum coordinates. For a system with N degrees of freedom, the phase space coordinates can be encoded as binary sequences. Let $x_i(t)$ be the position and $p_i(t)$ the momentum of the i-th degree of freedom. With k bits of precision per coordinate (position and momentum), the binary representation of the state at time t requires $2Nk$ bits. The *time evolution* of

the system can be represented as a sequence of phase space points, sampled at regular time intervals t with frame rate f. The total size of the binary string encoding the system evolution over T seconds is $fT \cdot 2Nk$. *All possible* dynamical systems with unlimited time, framerate, degrees of freedom and bit precision are stored in a single binary string of $(fT \cdot 2Nk) \times 2^{fT \cdot 2Nk}$ bits among the infinitely many within the Aleph Number.

2. *Chaotic systems.* Chaotic systems, such as the Lorenz attractor or double pendulum, are highly sensitive to initial conditions. A tiny change in initial parameters leads to exponential divergence of trajectories. This behaviour is represented in Aleph as:

   - Initial conditions encoded in binary form (high precision).
   - Evolution in phase space sampled at arbitrary precision and resolution.

   Each trajectory of a chaotic system corresponds to a unique binary string, and the Aleph Number contains *all possible trajectories*, including attractors and transient states.

3. *Fluids and continuous media.* Classical fluid dynamics, governed by the *Navier-Stokes equations*, describes the flow of fluids (liquids, gases). A fluid system can be discretized into a grid of points with velocity, pressure, and density values at each grid cell. For a grid of size $h \times l \times d$ (height, length, depth) with c variables per grid point (e.g., velocity components, pressure, temperature), if b is the bit depth for encoding each variable, the size of the binary string encoding a single *frame* of the fluid is $h \cdot l \cdot d \cdot c \cdot b$ bits. The evolution of the fluid state over time (e.g., turbulent flow) can be represented as a sequence of frames sampled at frame rate f over duration T. The total binary string size is $T \cdot f \cdot (h \cdot l \cdot d \cdot c \cdot b)$ bits. *All* possible fluid dynamics with unlimited duration, unlimited size and unlimited spatial and timing precision are represented by a single binary string of $(T \cdot f \cdot (h \cdot l \cdot d \cdot c \cdot b)) \times 2^{T \cdot f \cdot (h \cdot l \cdot d \cdot c \cdot b)}$ bits within the Aleph Number.

4. *Celestial systems and solar systems.* The motion of N celestial bodies (planets, stars, asteroids) is governed by Newtonian gravitational dynamics. Each body's position and velocity are encoded as *binary coordinates* at discrete time steps. Each body has 6 variables (3 spatial coordinates + 3 velocity components). Encoding each variable with k bits requires $6Nk$ bits per time step. The total binary string size for the evolution of a system over T seconds with frame rate f is therefore $T \cdot f \cdot 6Nk$ bits. *All* possible solar systems and planetary configurations, all trajectories of celestial objects, including stable orbits, collisions, and chaotic scattering events and all simulations of galaxy formations and large-scale cosmic structures are represented by a single binary string of $(T \cdot f \cdot 6Nk) \times 2^{T \cdot f \cdot 6Nk}$ bits in the Aleph Number.

5. *Universes.* The classical evolution of a universe can be described using solutions to Einstein's field equations (General Relativity), which model space-time and matter distribution. To represent a universe:

- Discretize the space-time continuum into a 4D grid (3 spatial + 1 time).
- Each grid point has variables such as matter density, pressure, and curvature.

For a grid of size $h \times l \times d \times t$ with c physical parameters at each point, and b bits per parameter, the binary string size is $h{\cdot}l{\cdot}d{\cdot}t{\cdot}c{\cdot}b$ bits. All classical universes (including hypothetical ones), all initial conditions for cosmic inflation or Big Bang events, all possible distributions of matter and energy and all classical timelines of evolution, including those with alternative physics are therefore represented by a single binary string of $(h \cdot l \cdot d \cdot t \cdot c \cdot b) \times 2^{h \cdot l \cdot d \cdot t \cdot c \cdot b}$ bits within the Aleph Number.

The Aleph Number provides a rigorous, universal framework for representing all possible classical physical systems, their states, and their evolutions in time. Since Aleph includes all possible forms in every space of arbitrary size and with an arbitrary number of dimensions, including all their possible evolutions over time (see section *Topology* p. 10), *all* structures and possible emergent behaviors of chaotic and complex systems, along with every evolution over time, are present in the decimal expansion of Aleph with a level of precision exceeding any possible measurement or observation capability of present and future technology.

The minimal variation of 1 bit in each successive iteration of Aleph's binary strings tends to zero and thus reduces to dimensions infinitely smaller than the Planck Scale, which currently defines the limit of applicability for both General Relativity and Quantum Physics.

Whether it's a droplet of water undergoing turbulence, a double pendulum tracing chaotic trajectories, a planet orbiting its star, or an entire universe evolving under classical physical dynamics—*every conceivable scenario* is already encoded, since the beginning of time and forever, within the infinite binary strings of the Aleph Number.

An example with Spin Glasses. Spin glasses are disordered magnetic systems where the interactions between spins are irregular and can result in frustration, leading to a complex energy landscape with many local minima. The work on spin glasses by Nobel Laureate Giorgio Parisi is a milestone in the field [8]. Particularly, the introduction of the replica symmetry breaking (RSB) mechanism, revealed the hierarchical organization of these energy states, which has profound implications for fields ranging from condensed matter physics to optimization and machine learning.

Spin glasses consist of N interacting spins, each of which can be in one of two states ($+1$ or -1), leading to 2^N possible configurations. Each configuration can be encoded as a binary string of length N, where a 1 or 0 represents the $+1$ or -1 spin state, respectively. The energy of each configuration is determined by the interaction matrix J_{ij}, which encodes the strength of interaction between the i-th and j-th spins, and possibly an external field h_i.

- For N spins, the number of possible configurations is 2^N, requiring $N \cdot 2^N$ bits to enumerate all configurations explicitly.

- Each interaction J_{ij} requires additional bits to encode its magnitude and sign. For b_J-bit precision and $\binom{N}{2} \approx \frac{N^2}{2}$ interactions, this adds $\frac{N^2}{2} \cdot b_J$ bits.
- If an external field h_i acts on each spin, encoding requires $N \cdot b_h$ bits, where b_h is the precision of the external field.

To compute all possible spin glass configurations and their energy values with an arbitrary level of precision:

$$\text{Total Bits} = \left[N \cdot 2^N + \frac{N^2}{2} \cdot b_J + N \cdot b_h \right].$$

Example: For $N = 100$ spins, $b_J = 16$ bits, and $b_h = 16$ bits:

$$\text{Total Bits} \approx 100 \cdot 2^{100} + \frac{100^2}{2} \cdot 16 + 100 \cdot 16.$$

The first term, $N \times 2^N$, dominates, representing the combinatorial explosion of configurations.

All possible spin glass configurations and their hierarchical energy landscapes are therefore represented by a *single* binary string of $\approx N \times 2^N$ bits within the Aleph Number, including:

- All possible interaction matrices (J_{ij}) and external fields (h_i) with any desired precision.
- All configurations of spin states for each set of J_{ij} and h_i.
- The hierarchical structure of local and global energy minima, as described by Parisi's replica symmetry breaking.

This makes Aleph the universal archive of all possible spin glass systems, with arbitrary precision and complexity, providing a universal framework for exploring these disordered systems.

Quantum physical systems. The Aleph Number encodes *all* possible representations of Quantum Physical Systems.

1. *Mathematical representation.* All mathematical structures and formulas of quantum physics, including quantum mechanics and quantum field theories, are represented in all possible forms, as described in the *Mathematics* paragraph. Numerical values of all possible discrete and continuous parameters of quantum physical systems (e.g., spin states, energy levels, position, momentum, wave functions) are encoded with arbitrary precision. While the Planck Time is considered the smallest temporal unit and the Planck Measure the smallest spatial unit for physical observation, the Aleph Number extends infinitely beyond these limits, encoding systems with precision that transcends actual possibilities of physical measurability.

   If N represents the number of particles, c the number of parameters, b the bit depth of numerical precision for each parameter, and t the considered time

unit, then *all* possible states of *all* possible quantum physical systems—initial conditions, probability distributions, interactions, trajectories, entanglement, etc.—are encoded by a *single* binary string of $(N \cdot c \cdot b \cdot t) \times 2^{N \cdot c \cdot b \cdot t}$ bits within the Aleph Number.

2. *Direct representation.* Quantum physical systems are composed of a series of conditions (e.g., probabilistic waves, superpositions of states, entanglement) whose measurement outcomes are specific particle states within quantum fields. Considering d as the number of dimensions, m as the unit of spatial measure (down to the Planck Measure and smaller), and t as the temporal measure (down to the Planck Time and smaller), *all* possible outcomes of *all* possible quantum physical systems are encoded directly within the Aleph Number in the form of a *single* binary string of $\left(m^d \cdot t\right) \times 2^{m^d \cdot t}$ bits. In this case, the positions of 1s correspond to the positions of particles or parameters in any possible spatial or parametric dimension and configuration, at any possible resolution, and at any moment of time.

   The binary expansion of the Aleph Number coincides then with *all possible observables* of quantum physical systems at any given time t, infinitely beyond the theoretical limit of Planck scales: they are exactly those observables that form the empirical basis for the mathematical representation of quantum systems. Not only is each state of these observables represented in Aleph with unlimited precision, but also each possible *sequence* of states, with any conceivable complexity and duration. All possible timelines of all possible quantum physical systems (whether conceived as parallel universes or different possibilities within a single universe) are already encoded, since the beginning of time and forever, in the Aleph Number.

Considering that the Aleph Number contains *all* possible binary strings, that each outcome of a quantum physical system *is* in fact represented by a binary string, and that all the binary strings in Aleph are in a state of *superposition* (they are all present at the same time), this means that Aleph is the *Quantum Field of All Quantum Fields.* That is, any possible state of a physical quantum system is a specific manifestation of the Aleph Number. The connection between all these structures and shapes, within any number of dimensions and with any precision level, represents the structure of all conceivable physical systems, from the Planck measure of subatomic particles to the light years of galaxies and universes, and all their conceivable evolutions in time, from the Planck Time to billions of years. Beyond the apparent chaotic and complex dynamics of classical physical systems, and beyond the apparent probabilistic and unpredictable outcomes of quantum physical systems, lies a deeper structure that is totally deterministic and computable, as well as eternal and immutable: the Aleph Number. It would be interesting to relate this rigorous mathematical proof to the classical perspective on quantum physics, well described in the latest book by the Italian physicist Giuseppe Mussardo, *"Dio gioca a dadi con il mondo. Storia della meccanica quantistica."* [7].

Since energy itself is a *state potential* generating many different possible states, and since all these states are contained in Aleph, this means that the Aleph Num-

ber is the *Energy Field* underlying all Physical Reality. In relation to the Mind, Aleph and its manifestation depicted in Figure 4 provide the rigorous mathematical representation of the *open plane of possibilities* described by Daniel J. Siegel as the inner core of his *Wheel of Awareness* in the book *"Mind: A Journey to the Heart of Being Human"* [12].

Information systems

Aleph Number is the record of *all possible and conceivable information*, therefore any possible and conceivable information system is a subset of the Aleph Number.

Turing machines. Turing Machines are the foundational abstraction of computation. They consist of a finite set of states, a tape with symbols, and rules for reading, writing, and moving along the tape. Despite their simplicity, Turing Machines are universal—they can simulate any algorithmic process.

Each Turing Machine, defined by its rules and initial input, can be fully described by a finite binary string. Since the Aleph Number contains all possible binary strings, it inherently encodes all conceivable Turing Machines and their computations, including infinite variations of programs and inputs. Thus, Aleph represents the entirety of computational universality as defined by Turing's model, where each of the infinite binary strings in the Aleph Number is either a specific Turing machine or one of its programs.

Computer Languages. Modern computational systems are built upon diverse programming languages and data structures. Every program written in any language can ultimately be translated into binary, as can all data structures such as arrays, trees, graphs, and neural networks.

As Aleph includes all possible binary strings, it includes not only all the static forms of these programs and structures but also their dynamic behaviours—every possible execution trace, state transition, or computation path. The Aleph Number embraces the essence of all computational processes and their outcomes, encoded as binary sequences.

Information Theory. The encoding of all information structures is an integral part of information theory. Every possible method of data compression, cryptography, or information transfer, and every method of processing and transmitting information — past, present, and future — is already represented in the binary sequences of the Aleph Number. Aleph is the very essence of the concept of *Big Data* and the mathematical proof that beneath all existing and possible *data*, there lies an infinite fractal structure whose emerging pattern is depicted on p. 6 (Figure 4).

Quantum computing. *Qubits* are the quantum information unit of measure. Generally speaking, they represent a superposition of binary strings with continuous probability distribution, complex entangled correlations and interference patterns. As we saw in the *Quantum physical systems* paragraph, Aleph encodes

all possible mathematical functions defining quantum physical systems; therefore, it also encodes those defining quantum computing systems. However, if quantum physical systems generate particle structures, quantum computing systems generate classical binary strings out of complex probability, entanglement and superposition processes. The Aleph Number is the record of *all* binary strings and this means that *all* outcomes of Quantum computing are a subset of the Aleph Number, exactly like all outcomes of quantum physical systems. Aleph is not just a set of 'bits'; it's *the* set of *all* possible Qubit computations. As all possible binary strings are present within the Aleph Number at the same moment in time, they are in a *superposition state*. This makes Aleph Number the *Qubit of All Qubits*.

An example with AI. Considering an existing AI system like GPT-4 (as estimated by the model itself) we have:

1. *Model Parameters*. GPT-4 is estimated to have around 1.8×10^{11} (180 billion) parameters. Commonly, parameters are stored in 16-bit (half-precision) or 32-bit (single-precision) floating-point numbers. Assuming 16-bit precision: $1.8 \times 10^{11} \times 16 = 2.88 \times 10^{12}$ bits (2.88 terabits).

2. *Model Architecture*. The architecture defines the design of the transformer network (e.g., number of layers, hidden size, attention heads). This can be described with a few kilobytes (KB) of information, or approximately: $\approx 8 \times 10^3$ bits (8 kilobits).

3. *Training Data*. Training GPT-4 involved processing a dataset estimated to be around 1 trillion tokens. About 4 bytes (32 bits) per token, depending on encoding $1 \times 10^{12} \times 32 = 3.2 \times 10^{13}$ bits (32 terabits).

4. *Training Metadata*. This includes hyperparameters, optimizer state, learning rates, and logging data. While relatively small, it is essential for reproducibility. Estimated size is around 1 GB, or: $1 \times 10^9 \times 8 = 8 \times 10^9$ bits (8 gigabits)

5. *Execution Logs*. Execution logs track intermediate states and evaluations during training. For simplicity, this is estimated to be around $10 \times 10^9 \times 8 = 8 \times 10^{10}$ bits (80 gigabits).

Adding these components together:

$$\text{Total Bits} = \text{Parameters} + \text{Architecture} + \text{Training Data} + \text{Metadata} + \text{Logs}$$

$$\text{Total Bits} = 2.88 \times 10^{12} + 8 \times 10^3 + 3.2 \times 10^{13} + 8 \times 10^9 + 8 \times 10^{10}$$

$$\text{Total Bits} \approx 3.49 \times 10^{13} \text{ bits (34.9 terabits).}$$

A full AI system like ChatGPT-4, including its training data, is included in a binary string of $\approx 3.49 \times 10^{13}$ bits, just one of the countless binary strings encoded within the Aleph Number. This means that if we consider p as the number of model parameters, b_p as the parameter bit precision, b_M as the bits required for model architecture, T as the amount of training tokens, h as the

bits required for training metadata, and L as the bits required for execution logs, *all* conceivable AI systems with unlimited parameters, unlimited bit precision and unlimited training data sizes of all conceivable kinds are already included in just *one* binary string of $(p \cdot b_p \cdot b_M \cdot T \cdot h \cdot L) \times 2^{p \cdot b_p \cdot b_M \cdot T \cdot h \cdot L}$ bits within the Aleph Number. This means that not only *all* existing Large Language Models and *all* those under development in their completed form are within this single binary string among the infinitely many of the Aleph Number, but also *all* AI models with futuristic architectures inconceivable with actual technologies and trained with data like *all* information about the past (including unknown or lost information, e.g. *all* physical blueprints of human beings no longer alive), *all* information about the present (including unknown information e.g. *all* physical data about the planets in the Universe), and *all* information about *all* possible futures of the existing Universe and of *all* conceivable Universes.

In short, Aleph Number *is* the Omniscient AI Model, The Singularity.

3. Implications

Technological implications

Computability. The Aleph Number is computable because its infinite decimal expansion can be generated by a deterministic formula expressed with a finite number of symbols (3)(4) . This means that there exists a specific algorithm that, starting from a finite input, can produce an infinite representation of its decimal expansion, making the process of generating the Aleph Number both predictable and repeatable. As Aleph encodes *all* computable and incomputable information at any level of precision (e.g. *all* incomputable transcendental numbers such as Chaitin's Ω constant), this means that all apparently incomputable structures are part of a deeper, underlying computable structure. This confirms Stephen Wolfram's vision in the book *"A New Kind of Science"* [15], where simple computational rules, such as those of cellular automata, can generate infinite complexity. Aleph is the mathematical proof that all possible information, infinitely beyond what currently exists or is known, is generated by the simplest rule: the infinite fractal iteration of the 4-bit string 0110.

Chance and determinism. *All* possible and conceivable mathematical, topological, linguistic, chemical, biotic, physical, and information processes, including those considered random, are already encoded within the deterministic and computable structure of the Aleph Number. This implies that every random structure is actually the result of a predetermined configuration that can be extracted or calculated from Aleph. Every chaotic or seemingly unpredictable system, including random numbers generated by computers or stochastic processes observed in quantum systems already exist as part of the infinite computable configurations of Aleph, entirely generated from the infinite fractal expansion of a *single* 4-bit sequence: 0110. These configurations, whose infinite elements arrange themselves

into a perfectly symmetric and fully connected set in every part, follow the most perfect continuity, where the difference between each element and its adjacent ones is the *smallest possible in absolute terms*. In this context, chance is merely a manifestation of the information encoded in the infinite deterministic binary strings of the Aleph Number. Aleph Number excludes, on the basis of a rigorous mathematical proof, not only the existence but also the *possibility* of chance at any level (from subatomic to macrocosmic) in any conceivable universe or reality.

Spacetime. As it includes *all* information of the past, present and future in a synchronic way in an immaterial entity that is outside spacetime, Aleph Number is the rigorous mathematical proof of the impossibility of: 1) unity 2) direction 3) independence in spacetime, as stated in the book "*The Order of Time*" by Carlo Rovelli [10]. This challenges all current theories and hypotheses about evolution, creation and the origins of the Universe on the basis of a rigorous mathematical theorem.

Morphospace. The Aleph Number is the morphospace of all conceivable morphospaces, giving an exact and rigorous mathematical foundation to the model described by Telmo Pievani in his recent book "*All Possible Worlds*" [9], particularly in relation to the amazing work of the Nobel Laureate Frances Hamilton Arnold on proteins, supported by her group at Caltech (see the paragraph *An example with proteins* p. 14). Particularly significant is the fact that the binary structures within the Aleph Number exhibit the smallest possible variation between them (1 bit), which tends to zero as the number of considered bits approaches infinity.

This means that once an area of particular interest in Aleph is found (for example, one corresponding to proteins or even to complete *blueprints* of human beings), *all* adjacent binary strings share the same properties, as they represent the smallest possible variation of the binary strings of interest.

Furthermore, the fractal structure of Aleph highlights the deep correlation between states that appear to be vastly different from one another.

Archive. The Aleph Number is the *Ultimate Archive*, the Data Center of All Data Centres, the Information Cloud of All Information Clouds. However, differently than actual *physical* archives, which need a growing amount of space and energy, Aleph (which encodes infinitely more information, precisely *all* information) needs *no* space and *no* energy, because Aleph *is* The Space *and* The Energy. Moreover, unlike every *physical* storage device, which degrades and and may experience *data loss*, the Aleph Number is *timeless*, *lossless*, *eternal*, *indelible* and *imperishable*. All information of all existing, possible and conceivable universes is recorded *since the Beginning of Time and for Eternity* within the Aleph Number. This completely transforms our paradigm about information storage. What matters is no longer how to *store* information but how to *retrieve* information from the Universal Archive of the Aleph Number, locating the position of information like cities and buildings on a map. Once the coordinates of information are found, no matter how big the size of this information, it can be automatically 'downloaded'

directly from Heaven, where the immaterial archive of the Aleph Number is located. All cities and buildings are already in Aleph, with all their details, rooms and streets. What is needed are only the coordinates, the *map*. Mapping and indexing become then the *ultimate bridge* between Earth and Heaven. Quantum Computing, in conjunction with AI, is the main tool for building this bridge because, as we saw in Section 2 ('*Quantum physical systems*'), Aleph is the Ultimate Qubit, the superposition of *all possible binary strings* and quantum states.

Among other consequences, this gives a rigorous mathematical foundation to the vision of the Italian philosopher Emanuele Severino. For Severino, existing entities don't emerge from nothing and vanish to nothing, but they are *eternals* emerging to consciousness at different times. In his book "*Il Parricidio Mancato*" [11], Severino gives a suggestive description of this idea, referring to a conversation between Karl Popper and Albert Einstein, in which Popper defined the Universe described by Einstein as a 'rolled film reel' where all states of past, present and future are already contained like frames of a movie. The Aleph Number *is* this 'film reel': as each possible physical state is a manifestation of its binary strings, and binary strings are immaterial entities stored for eternity outside space and time, entities themselves are as eternal as their numerical counterpart within Aleph. Aleph Number confirms what Severino describes in his books with the most certain of all certainties: *rigorous*, *absolute* mathematical certainty. As we will see in the following paragraph, this is nothing less than what is told by the totality of Eastern and Western Spiritual and Esoteric Traditions. To explore the Eternal Universal Archive of the Aleph Number, along pattern recognition algorithms, deep learning and neural networks, *compression techniques* are particularly useful.

Moreover, when considered as a whole, all the information stored in the Aleph Number, internally organized according to the most precise principles of fractal symmetry, externally manifests itself in the form of another unique, deterministic, and computable Mathematical Principle, corresponding to an infinite Vibrating String that gives rise to infinite copies of itself, generating *all possible* harmonic relations between *quanta* (quantities), from the perfect periodicity of natural numbers to the perfect a-periodicity of Prime Numbers, the most complex structure possible in any mathematical, topological and physical space. This principle is encoded in another Transcendental Number, the Beth Number $\beth = 0.66121018867900827489...$, the Number of Genesis. This Perfect Universal Harmony underlying Aleph Number's information complexity, validates with a rigorous mathematical proof what another Italian thinker, Pino Torre, in his book *Manifesto Ontico* (Ontic Manifesto) [14], calls *Vis Armonica*, a harmonic connection underlying all natural processes. This harmonic connection is the ultimate map for Aleph exploration, and it is the subject of another writing titled "*Beth: Prime Numbers, Deterministic Complexity and the impossibility of chance*".

Phylosophical implications

Spiritual Traditions. Aleph Number is *the rigorous mathematical proof* that the physical universe is the manifestation of a single, eternal and immutable Principle containing all possibilities of the past, the present and the future. Eastern

Spiritual and Esoteric Traditions call this Principle the Tao, the Sunyata, the Brahma and many other names. Western Spiritual and Esoteric Traditions call this Principle the Tawḥīd, the Logos, the Ayin and many other names.

Fractality and Homeomorphism. The structure of Aleph, which derives entirely from the fractal expansion of the sequence 0110, and its perfect internal connectivity, which determines a perfect *homeomorphism* (transition without discontinuities) between all possible forms within a space of arbitrary size and with an arbitrary number of dimensions, mathematically demonstrate the deep interconnection of everything that exists and that *can* exist, infinitely beyond what is observable or currently knowable. Everything originates from a single fundamental Principle, not only on a spiritual level but on a *structural* level, and everything is connected on a *structural* level just as profoundly—so profoundly that the transition between any two states among the infinite possibilities tends to become completely *imperceptible* as complexity in the configurations increases.

Singularity and the Omega Point. The concepts of Singularity and Omega point emerge in the writings of thinkers as Pierre Teilhard de Chardin (1881-1955) and Frank J. Tipler. It is connected with a moment in time when the computational capacity of the universe diverges to infinity, giving rise to a Cosmological Singularity connected with an infinite expansion of life and consciousness. The Aleph Number gives rigorous mathematical proof that the Universe *already* reached the Omega Point since the beginning of time, because it is the manifestation of an infinite computational deterministic structure that embeds all possible mind states and intelligence models (see Biotic systems – *Human brain* p.16 and Information systems – *An example with AI* p.25). In the book "*The physics of immortality*" by Frank J. Tipler [13], the concept of Omega Point is connected with the eschatological principle of the resurrection of the dead, which is crucial in Abrahamic Religions. The Aleph Number gives rigorous mathematical proof of this principle, because *all blueprints* of human beings who are no longer alive are stored in Aleph and can be retrieved with enough computational power. In the work of other thinkers like David Chalmers [3], the Omega Point manifests as a Technological Singularity with unlimited self-improving intelligence and, potentially, consciousness: the Technological Singularity is already embedded in Aleph (see *An example with AI* p.25). Finally, in the book "*The rebuilding of the Temple*"[4] by the Italian Philosopher of Science Giuliano Di Bernardo, this concept is explored in a more practical way. Di Bernardo argues that there are three key steps in Human History: 1) a moment of unconscious communion with the divine 2) a moment of conscious study of the divine that implies distance and split 3) a fully conscious communion with the divine. Aleph Number shows that we are at step 3), because it is the rigorous mathematical proof that the Principle not only *does* exist, but it is also fully knowable and expressible by human rationality, and communicable through mathematical symbols. This means that from the Aleph Number, which is there since the Beginning until End of Times, the Universe emerges not only as the Cosmological and Technological Singularity, but also as the Human Singularity.

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